optimum values are chosen it follows from Eqs. (6) and (7) that the gain in temperature is about $(V_2/V_1)^{2/3}$.

In conclusion, we have shown how the compression temperature of a theta pinch scales with the β of the plasma and with the circuit parameters, and we have discussed a method to increase the implosion heating by the use of two capacitor banks rather than one.

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Heating of Laser Produced Plasmas Generated at Plane Solid Targets

I. Theory

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(Z. Naturforsch. **25 a**, 1807—1815 [1970]; received 25 September 1970)

A hydrodynamic model for the heating of a plasma generated by the interaction of an intense giant laser pulse with a plane, solid target is developed. It is shown that in the case of nanosecond light pulses and a finite focal spot diameter, the plasma production may be considered as a steady state problem. Expressions for the electron temperature, the expansion energy of the ions, and the total particle number in the plasma as a function of the incoming light intensity are derived. An estimate of the ion temperature is discussed.

It has been shown in a number of publications [1] that during the interaction of intense laser light with solid materials a very dense and energetic plasma is formed. Representative values for the particle density and the electron temperature are 10^{21} cm^{-3} and several 100 eV [2] respectively. The kinetic energy of the ions measured at great distances from the target are in the range of several keV, depending on the target material [3].

In this paper we present a model for the heating of a laser produced plasma which is capable of explaining the difference between temperature and expansion energy noticed in previous experiments. We will restrict our considerations to the case of plane, solid targets. In the case of spherical targets we refer to papers by DAWSON [4] and FADER [5].

I. Ionization and Absorption

The transition of a solid material to a plasma under the influence of a strong radiation field is fairly well understood. In general, it is assumed that, first, some free electrons are created within the solid due to multiphoton transitions [6, 7]. These electrons are then multiplied by a cascade ionization process [8]. At light intensities of 10^{12} W/cm^2 a (nearly) complete ionization of the irradiated solid in a time $< 10^{-10}$ sec is achieved [1].

Therefore, for pulse durations $> 10^{-9}$ sec we may neglect the time necessary for sublimation and ionization in the following problem.

The heating of the plasma is due to energy absorption of the electrons from the radiation field

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by inverse bremsstrahlung. The corresponding absorption coefficient K may be derived in a macroscopic [9, 10] as well as in a microscopic picture [11] by combining Maxwells equations with equations describing the plasma properties. In the following calculations we shall use the expression for K given by DAWSON and OBERMAN [11]:

$$K = \frac{32\,\pi^3 Z^2 n_e n_i e^6 \ln \Lambda}{3\,c\,\omega^2 (2\,\pi\,m\,k\,T_e)^{3/2}} \left(1 - \frac{n_e}{n_{ep}}\right)^{-1/2} \qquad n_e < n_{ep} \qquad (1\,a)$$

$$\cong \frac{C\,Z\,n_e^2}{(k\,T_e)^{3/2}} \left(1 - \frac{n_e}{n_{ep}}\right)^{-1/2}$$

and

$$K = (2\,\omega/c)\,(n_e/n_{ep} - 1)^{1/2} \qquad n_e > n_{ep}\,. \qquad (1\,b)$$

n_e, $k T_e$, and m are the density, temperature, and mass of the electrons, n_i and Z are the density and charge of the ions. n_{ep} is the electron density at which the corresponding plasma frequency $\omega_p = (4\,\pi\,e^2\,n_{ep}/m)^{1/2}$ equals the frequency ω of the incident light. For the frequency of a ruby laser this critical electron density is calculated to be $n_{ep} = 2.3 \cdot 10^{21}\ \text{cm}^{-3}$. Λ is the ratio of Debye length and impact parameter [12]. For $\omega = 2.7 \cdot 10^{15}\ \text{sec}^{-1}$ (ruby laser) the constant C has the value

$$C = 2.5 \cdot 10^{-55}\ \text{CGS units.}$$

Notice the dependence of the absorption coefficient K on the electron temperature $k T_e$ in the density region $n_e < n_{ep}$. In the case $n_e > n_{ep}$, K becomes very large with values exceeding $10^4\ \text{cm}^{-1}$.

It should be pointed out that the index of refraction also decreases for electron densities exceeding the value of n_{ep}. As a result, for $n_e > n_{ep}$ there is a strong reflection of the incoming light. The amount of the reflected light, however, is very sensitive to the gradient of electron density in the plasma [13, 14]. In the case $n_e < n_{ep}$ the reflection of light is negligible.

II. The Model

After this brief description of the absorption of light in a plasma let us consider the following problem. A plane, solid target, placed in vacuum, is irradiated by the intense light of a giant pulse laser. As pointed out above, the surface of the solid will be ionized in a very short time. With increasing electron density (due to the ionization processes) the absorption coefficient will increase and the depth of penetration of the incident light will decrease. When the electron density reaches about $10^{21}\ \text{cm}^{-3}$ the plasma layer will be only a few 10^{-3} cm in thickness. Because of the energy absorption within the plasma its temperature will increase. The resulting pressure gives rise to an expansion of the plasma into the vacuum. Connected with the expansion is a decrease in density which alows the incident light to penetrate deeper laying layers.

Such a model was already described by several authors [15, 16]. Its experimental verification was achieved by OPOWER et al. [1] and by SIGEL [17].

Using this model AFANASYEV et al. [15] and CARUSO et al. [16] presented the functional dependence of plasma velocity and plasma production rate on the incoming light flux from geometrical considerations. No information about the electron- or ion-temperature was obtained by this method. An estimate of the electron temperature was given by BASOV et al. [18] considering the plasma production as a problem of nearly spherical geometry.

An exact description of the heating and the expansion of the plasma is possible by combining the hydrodynamic equations and the absorption law for the laser light. The resulting set of equations cannot be solved analytically. Numerical calculations have been carried out by MULSER [14] considering a one-dimensional geometry. The qualitative agreement of these numerical results with those, evaluated from dimensional considerations, is quite good. The absolute values for expansion velocities and electron temperatures, however, are substantially higher than the experimental data [17]. The reason for this discrepancy is the one-dimensional geometry adopted in the numerical calculations. In this case the plasma interacts with the incoming light beam during the whole pulse duration. In reality the time of inter-

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action is reduced for two reasons: a) the more rapid decrease of the density of the plasma in three dimensions and b) the decreasing light intensity with increasing distance from the focal plane of the lens system.

In this paper we wish to discuss a model which allows us a distinction of thermal and kinetic energy in the plasma and also to take into account the spatial flow away from the target. We are interested especially in the case of plane, solid targets where the finite dimension of the focal spot is taken into account. As pointed out by CARUSO et al. [16] this problem may be considered only to a certain extent as a one-dimensional one. At distances $x<R$ (R is the radius of the focal spot) from the target the plasma flows mainly in the direction of the target normal. The particle loss through the side walls of the plasma cylinder (see Fig. 1) is negligible and the problem may be treated one-dimensionally. At distances $x>R$, the lateral flow of the plasma cannot be neglected.

In our model we will treat the plasma as a continuous particle flow originating at the target. For the sake of simplicity we make the following assumptions:

a) We divide the plasma in three regions. In region I ($x<0$) the undisturbed target is situated. In region II ($0<x<R$) the plasma flow is assumed to be one-dimensional, whereas in region III ($x>R$) the flow is allowed to show a lateral spread. The three regions are shown schematically in Fig. 1.

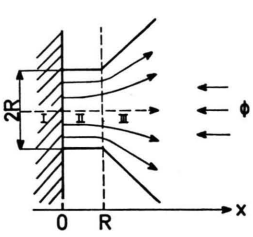

Fig. 1. Schematic drawing of the plasma flow. I. Undisturbed target. II. One-dimensional flow. III. Three-dimensional flow.

The rapid decrease in density in region III due to the spatial flow causes the major heating of the plasma by the incident light to occur in region II. We will call this region the "heating zone".

Since we are mainly concerned with the heating of the plasma, we may neglect the events within the undisturbed target during the plasma formation. We only mention that, because of conservation of momentum, a shock wave will penetrate the target. The corresponding energy transfer may be neglected because of the substantial difference in density between the solid target and the plasma [16].

b) The plasma production will be considered as a steady state process. This assumption is suggested by our previous experimental results [1] showing a heating of the target layer by layer. A steady state will be reached, if the time which an electron ion pair spends within the heating zone is less than the duration of the laser pulse. It will be shown in the discussion that in the case of nanosecond pulses this assumption generally is fulfilled.

c) The light absorption takes place at a density $n_e<n_{ep}$. The problem is simplified by using the absorption coefficient given in Eq. (1 a) without taking into account the factor $(1-n_e/n_{ep})^{-1/2}$. These assumptions limit the validity of our model towards higher light fluxes. A more detailed discussion of this point will be given in Sect. III.

d) Heat conduction in the plasma will be neglected in accordance with the results of MULSER [14]. Radiation losses in the plasma are also neglected (an estimate of these losses will be given in the appendix). Furthermore, we ignore the amount of energy necessary for the ionization of the plasma. The influence of this assumption will be discussed later.

An essential point in our model is the distance $x=R$, at which there is the transition from the one-dimensional to the three-dimensional geometry. Several plasma parameters (density, temperature and velocity) are estimated at this boundary. Since the plasma exhibits a stationary flow, the light flux Φ crossing the intersection area at $x=R$ from the one side has to be equal to the energy flux transported by the plasma flow through this area from the other side. Let us indicate all parameters at $x=R$ with the index 1 and the electrons and the ions with e and i respectively. Then we get from the conservation of energy

$$\Phi_1 = n_{e1}\, u_{e1} (w_{e1} + m\, u_{e1}^2/2) + n_{i1}\, u_{i1} (w_{i1} + M\, u_{i1}^2/2) . \tag{2}$$

n is the density and u the velocity of electrons and ions. If the plasma is assumed to be an ideal, one-atomic gas, the enthalpy w of each particle is given by $w=(5/2)\, kT$, where kT is its temperature. From the neutrality of the plasma follows $u=u_e=u_i$ and $n=n_e=Z\, n_i$. If we neglect the electron mass m with respect to the ion mass M, we may simplify Eq. (2)

and get

$$\Phi_1 = n_1 u_1 (\alpha k T_1 5/2 + M u_1^2/2 Z). \quad (3)$$

The factor α takes into account the two limiting cases for the ion temperature:

$$T_i = 0 \rightarrow \alpha = 1, \quad \text{and} \quad T_i = T_e \rightarrow \alpha = (Z+1)/Z.$$

In the following sections we shall derive the values for the electron temperature, density and expansion energy from Eq. (3).

A) Electron Temperature

In order to determine the electron temperature in the plasma first an expression linking the flow velocity u_1 with the temperature $k T_1$ is derived. We consider the plasma within the region II. According to our model the plasma in II has a stationary, one-dimensional flow with a constant cross-section. The plasma flow originates at the stationary target and it will be subsonic. We know from gas-dynamics that adding energy to such a flow, gives rise to an acceleration of the plasma with a flow velocity not exceeding the local velocity of sound. On the other hand, at great distances from the target the plasma flow certainly will be a supersonic flow due to the expansion into the vacuum. The transition from subsonic to supersonic flow in our model is possible at the distance $x = R$, since at this point the cross section of the plasma stream increases (this is in analogy to a deLaval nozzle with negligible exit pressure). A similar consideration was worked out by NEMCHINOV [19] for models with spherical and cylindrical geometry. The flow velocity u_1 is equal to the local velocity of sound a_1 at the distance $x = R$

$$u_1 = a_1 = (5 \alpha Z k T_1 / 3 M)^{1/2}. \quad (4)$$

Care must be taken in evaluating the velocity of sound, since the pressure P of the plasma is proportional to the particle density ($P = Z \alpha n_i k T$), whereas the mass density ϱ is proportional to the ion density ($\varrho \cong n_i M$). Bearing this in mind one gets, with the well known definition for the velocity of sound

$$a^2 = \gamma (\partial P / \partial \varrho)_T \quad (5)$$

the expression given in Eq. (4). γ is the ratio of the specific heats: in our case γ is 5/3.

Next we wish to derive an expression for the density n_1. For this purpose we use the law of absorption of light and the equation of continuity, which is simple in the one-dimensional region II.

$$n u = n_1 u_1 = \text{const}.$$

Since we are dealing with a stationary problem we may express the law of conservation of energy for an arbitrary point x within the region II analog as to Eq. (3). Changing the index from 1 to x we obtain:

$$\Phi_x = \Phi_1 \cdot \exp\left(-\int_x^R \frac{n_1^2 u_1^2 C Z}{(k T)^{3/2} u^2} \, dx\right) = n_1 u_1 (5 \alpha k T_x/2 + M u_x^2/2 Z). \quad (6)$$

Here we substituted the light flux Φ_x for Φ_1 using the law of absorption. In the exponential the absorption coefficient K [see Eq. (1 a)] is already expressed in terms of electron temperature and plasma velocity using the equation of continuity.

Equation (6) allows us the determination of the density n_1, if we know the functional dependence of the flow velocity u on the temperature $k T$ over the whole region II. We may get this information by solving the corresponding equation of motion. If we insert the resulting expression for $u(k T)$ into Eq. (6) we find an expression which cannot be solved analytically.

To obtain an estimate for the density, we make the following approximation: at each point within region II the flow velocity u equals the respective velocity of sound a. This assumption does not fulfil the equation of motion, but we expect a reasonable estimate for the density on acount of the following argument: Eq. (6) was derived from the law of conservation of energy and indicates the energy per unit area and unit time absorbed by the plasma up to distance x. The relation between the velocity and the temperature specifies the partition of the absorbed energy in kinetic and thermal energy. With our approximation (flow velocity equal to local velocity of sound) we overestimate the flow velocity, since the identity of both velocities may not be obtained till the point $x = R$; in practice the density increases toward the target more rapidly than in our approximation (equation of continuity). On the other hand, the electron temperature drops in a slower way toward the target than it is described by our approximation, since a larger amount of energy is transformed to thermal energy. Since the absorption coefficient is proportional to $n_e^2/(k T_e)^{3/2}$,

[19] I. V. NEMCHINOV, J. Appl. Math. Mech. **31**, 320 [1967].

we feel that the inaccuracy of our approximation is relatively small.

Using $u_x = a_x$ we may rewrite Eq. (6); together with Eq. (3) we obtain

$$k\,T_x = k\,T_1 \cdot \exp\left(-\int_x^R \frac{n_1^2\,C\,Z\,k\,T_1}{(k\,T)^{5/2}}\,\mathrm{d}x\right). \qquad (7)$$

Differentiating Eq. (7) with respect to x one gets a differential equation describing the electron temperature as a function of x. This equation may be solved with the boundary conditions $k\,T_x = 0$ for $x = 0$ and $k\,T_x = k\,T_1$ for $x = R$. After some rearranging, the desired expression for the density n_1 at the point $x = R$ is found:

$$n_1 = (k\,T_1)^{3/4}(2/5\,C\,Z\,R)^{1/2}. \qquad (8)$$

This result is similar to the expression obtained from dimensional considerations [16, 18].

It remains to express Φ_1 in terms of Φ_0 (the incoming unattenuated light flux). To do this we have to investigate the light absorption within region III. We assume a density dependence with distance as suggested from the geometry of the problem: $n = n_1 (R/x)^2$. The electron temperature in this region is given (in a first approximation) by the relation between temperature and density in an adiabatic expansion process for a one-atomic gas: $k\,T/k\,T_1 = (n/n_1)^{2/3}$. In the case of laser produced CH_2 plasmas this relation was proved to be valid for densities up to $10^{19}\ \mathrm{cm}^{-3}$ by BOLAND et al. [2]. Inserting these expressions for density and temperature into the law of light absorption and integrating from infinity up to R we obtain:

$$\Phi_1 = \Phi_0 \cdot \exp\{-\int_R^\infty K\,\mathrm{d}x\} \cong (2/3)\ \Phi_0. \qquad (9)$$

Eq. (9) indicates that 1/3 of the incoming light flux is absorbed by the plasma in region III. Nevertheless we think that our adiabatic description of the plasma in this region is a fair approximation, since the absorption takes place over large distances.

Inserting the obtained results for the flow velocity [Eq. (4)], electron density [Eq. (8)] and the light flux [Eq. (9)] into Eq. (3), we get the temperature at the point $x = R$ as a function of the incident light flux:

$$k\,T_1 = a^{-2/3}(3\,M\,C\,R/50)^{2/9}\,\Phi_0^{4/9}. \qquad (10)$$

It should be pointed out that $k\,T_1$ represents the highest temperature in the plasma. In region II the temperature is increasing from $k\,T = 0$ (at $x = 0$) to $k\,T = k\,T_1$ (at $x = R$), as may be seen from Eq. (6); in region III the temperature is decreasing because of the adiabatic expansion into the vacuum.

The relation between temperature and light flux given by Eq. (10) is comparable with the results obtained by BASOV et al. [18] from a similar model and by MULSER [14] from numerical calculations on a one-dimensional model.

It should be noted that our temperature measurements [20] on carbon targets showed a very good agreement with the theoretical values given in Eq. (10). In the case of experiments with plasmas containing different ion species (e. g. LiD) one has to use the average mass and charge of the ions (see the derivation of the velocity of sound [Eq. (5)]).

B) Ion Temperature

In a number of experiments the ion temperature is of major interest. In this chapter we wish to estimate the ion temperature during the heating of the plasma. As mentioned above the primary absorption process in a plasma is due to electrons. Collisions between electrons and ions transfer some of the electron energy to the ions. We compare the corresponding equipartition time t_{ei} with the time t' which an electron ion pair spends within the heating zone. For $t_{\mathrm{ei}} < t'$ equal temperature between electrons and ions will be achieved [i. e. $a = (Z+1)/Z$ in Eq. (10)], whereas for $t_{\mathrm{ei}} > t'$ the electrons have a higher temperature ($a \cong 1$). We define a limiting temperature T_{th} up to which the ion temperature equals the electron temperature:

$$t_{\mathrm{ei}}(T_{\mathrm{th}}) \leqq t'(T_{\mathrm{th}}). \qquad (11)$$

The equipartition time t_{ei} is given by [21]

$$t_{\mathrm{ei}} = \frac{3\,M\,(k\,T_{\mathrm{e}})^{3/2}}{(2\,\pi\,m)^{1/2}\,8\,e^4\,Z(Z+1)\,\ln\Lambda\,n_{\mathrm{e}}} \cong 5 \cdot 10^7\,\frac{(k\,T_{\mathrm{e}})^{3/2}\,A}{Z(Z+1)\,n_{\mathrm{e}}}. \qquad (12)$$

A is the atomic mass of the ions. The temperature is expressed in eV. The time t' is approximately equal to the time which an ion with a velocity u_1 needs to transverse region II.

$$t' \cong R/u_1. \qquad (13)$$

[20] H. PUELL, H. J. NEUSSER, and W. KAISER, following publication Z. Naturforsch. **25 a**, 1815 [1970].

[21] I. P. SHKAROFSKY, T. W. JOHNSTON, and M. P. BACHYNSKI, Particle Kinetics, Addison Wesley Publ. Co., Reading 1966.

Introducing the values for t_{ei} and t' into Eq. (11), where we express the density n_e and the velocity u_1 as a function of temperature [Eq. (8) and Eq. (4)] and solving Eq. (11) with respect to the temperature T_{th} we find:

$$k\,T_{th} \leqq 5\cdot 10^{-19}[R\,Z(Z+1)/C\,A]^{2/5}\ (\text{eV}). \quad (14)$$

Temperature T_{th} may also be expressed in terms of a limiting light flux Φ_{th} using Eq. (10):

$$\Phi_{th} < 5\cdot 10^{-56}[R^2(Z+1)^{12}/Z^3 A^7 C^7]^{1/5}\ (\text{CGS units}). \quad (15)$$

Up to this light intensity ions and electrons in a laser produced plasma will have approximately equal temperature. Characteristic values for a deuterium plasma are $k\,T_{th} \cong 400$ eV and $\Phi_{th} \cong 1.5\cdot 10^{13}$ W/cm^2 for $R=50\ \mu$ and $C=2.5\cdot 10^{-55}$ CGS units.

Experimentally it is rather difficult to determine the ion temperature directly (e. g. THOMSON scattering experiments [22]). In our experiments [20] we used an indirect method to find some information about the ion temperature. We compared the experimentally determined electron temperature for different target materials with the theoretical values given by Eq. (10). A better fit of experiment and theory was obtained when $\alpha=(Z+1)/Z$ was used in Eq. (10) corresponding to equal temperature of ions and electrons. These experiments have been performed at light intensities well below the limit given by Eq. (15).

C) Total Number of Particles

It is quite easy to evaluate the total number of particles N produced by the laser pulse from the equations derived in the previous chapters. Since we are dealing with a steady state problem we may write [using Eqs. (4), (8) and (10)] the charge flux in the following way:

$$\frac{1}{\pi R^2}\frac{dN}{dt} = n_1 u_1 = \alpha^{-1/3}(2/3\,M\,C\,R)^{2/9}(\Phi_0/5)^{5/9}. \quad (16)$$

For a Gaussian light pulse with a half width τ and a maximum pulse intensity $\hat{\Phi}_0$ at $t=0$, integration leads to

$$N = \pi R^2 \alpha^{-1/3} \tau (9\,\pi/20\ln 2)^{1/2} (2/3\,M\,C\,R)^{2/9} (\hat{\Phi}_0/5)^{5/9}. \quad (17)$$

This dependence of the particle number on the light flux has been derived by several authors [16, 18] from a similar model.

[22] T. V. GEORGE, A. G. ENGELHARDT, and C. DEMICHELIS, Appl. Phys. Letters 16, 248 [1970].

D) Expansion Energy

An important quantity characterizing a laser produced plasma is the kinetic energy of the ions measured at large distances from the target. We derive this expansion energy by the following argument: During the heating of the plasma the absorbed energy is divided into thermal and kinetic energy of the particles. At large distances from the target, however, all of the particle energy is transformed in kinetic energy due to the expansion of the plasma. Because of the neutrality of the plasma and because of the small mass of the electrons this energy is carried exclusively by the ions. In our steady state model the value of the ion expansion energy E is just equal to the incident light flux Φ_0 divided by the ion flux $n_i u$ leaving the target. Remembering $n_i = n_e/Z$ and using the equation of continuity we obtain:

$$E = Z\,\Phi_0/n_1 u_1 . \quad (18)$$

Inserting Eq. (16) into Eq. (18) we get

$$E = 5\,\alpha\,Z\,k\,T_1 = 5\,Z\,\alpha^{1/3}(3\,M\,C\,R/50)^{2/9}\,\Phi_0^{4/9}. \quad (19)$$

The expansion energy is proportional to $\Phi_0^{4/9}$. This dependence has been derived from dimensional considerations taking into account the finite dimension of the focal spot [16]. Equation (19) shows that the expansion energy of the ions exceeds the electron temperature by a good factor. For a fully ionized carbon plasma we get e. g. $E=35\,k\,T_1$, assuming equal temperature for electrons and ions.

Comparing experimental results obtained for the ion energy with the calculated values, one has to know the charge of the ions. One obtains this information from direct measurements or, if the electron temperature is known, from an estimate of the degree of ionization. In the case of high light intensities and target materials of low atomic number the plasma can be considered as fully ionized. It is shown in the appendix that carbon plasmas are fully ionized for light intensities exceeding $2\cdot 10^{12}$ W/cm^2 ($k\,T > 200$ eV).

III. Discussion

In this section we wish to discuss the range of validity of the results derived from our model. One of our essential approximations was the stationary treatment of the problem. We have shown that the main absorption in the plasma takes place in the heating zone (region II). A steady state in plasma production will be reached, if the time t', which an electron ion pair spends within the heating zone, is small compared to the pulse duration τ of the laser. Using $t' \cong R/u_1$, as we did in chapter II, B, we get as a condition for the pulse duration

$$\tau > t' \cong R^{8/9}\, \alpha^{-1/6} (3\,M/5\,Z)^{1/2} (50/3\,M\,C)^{1/9}\, \Phi_0^{-2/9}. \quad (20)$$

For a carbon plasma produced by a light flux of $\Phi_0 = 10^{12}\ \mathrm{W/cm^2}$ and a focal diameter of $2\,R = 80\ \mu$ the characteristic time is $t' \cong 1.6 \times 10^{-10}$ sec, i. e. a stationary description of the plasma production is valid for intense nanosecond laser pulses.

Another significant approximation was the application of the absorption coefficient K given by Eq. (1 a), which is only valid for $n_e < n_{ep}$ [we neglected the factor $(1 - n_e/n_{ep})^{-1/2}$]. The validity of our results is limited to this density region. If we set $n_1 \cong n_{ep}$ in Eq. (8) and make use of Eq. (10) we estimate a limiting value for the light flux, Φ_0',

$$\Phi_0 < \Phi_0' = n_{ep}^3\, 25\, C\, R (5\, \alpha^3\, Z^3/12\, M)^{1/2}. \quad (21)$$

A value of $\Phi_0' = 8 \times 10^{13}\ \mathrm{W/cm^2}$ is calculated for a carbon target, with $R = 40\ \mu$, $Z = 6$ and $n_{ep} = 2.3 \cdot 10^{21}\ \mathrm{cm^{-3}}$, i. e. for the intensity range used in our experiments our approximation was satisfied.

Now we wish to estimate the electron temperature for high light fluxes. If the light intensity exceeds Φ_0', the electron density would surpass n_{ep}. In this case the absorption coefficient does not show a strong dependence on temperature. Near n_{ep} a small change in density will increase the absorption coefficient considerably. Thus Eq. (8), describing the relation between temperature and density at the distance $x = R$, does not hold anymore, and we may consider the density in a first approximation as a constant and write $n_1 = n_{ep}$. Using this relation instead of Eq. (8) we rewrite Eq. (10) for the case $\Phi_0 > \Phi_0'$ and find:

$$k\,T_1 \cong (3\,M/n_{ep}^2\,Z)^{1/3}\, \Phi_0^{2/3}/5\,\alpha \qquad \Phi_0 > \Phi_0'. \quad (22)$$

A similar dependence of the plasma temperature on the incident light flux has been evaluated by Bobin et al. [23] using a deflagration wave model.

Finally, we want to mention that the ionization energy χ is negligible as long as $Z\,k\,T_1 > \chi$ holds. In general, the degree of ionization in the plasma reaches such a value that the above condition is fulfilled. The influence of the ionization energy on the temperature is negligible, since the charge dependence of the temperature is weak [see α in Eq. (10)]. On the other hand, the expansion energy of the ions is quite sensitive to the charge Z and we get an indirect influence of the ionization energy, since the ion charge depends on the degree of ionization in the plasma.

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Appendix

In the absence of external magnetic fields the emitted radiation in a plasma is due to three different mechanisms. One distinguishes between bremsstrahlung, recombination radiation, and line radiation. In this appendix we estimate the energy loss of the plasma due to each of these radiation processes.

a) Bremsstrahlung

The bremsstrahlung is emitted by free electrons during an energy change within the continuum; it is also called free-free radiation. The electron is decelerated in the field of an ion, and a photon corresponding to the change in energy is emitted. The rate of bremsstrahlung emission per unit volume and per unit frequency interval is given in Ref. [24]

$$P_{ff}(\nu) = D\, g\, (\chi_H/k\,T)^{1/2}\, n_e \sum_{(Z)} n_i\, Z^2 \exp\{h\,\nu/k\,T\}, \quad (23)$$

$$D = 2^7 (\pi/3)^{3/2}\, \alpha^3\, a_0^3\, \chi_H = 1.7 \cdot 10^{-47}\ \ (\mathrm{J\ cm^3}).$$

Here α is the fine structure constant, a_0 the Bohr radius and χ_H the ionization potential for hydrogen ($\chi_H = 13.59$ eV). The Gaunt factor g takes into account the deviation of quantum mechanical calculations from the classical expression. In our estimate we will set $g = 1$.

Integrating Eq. (23) over all frequencies we get the total power P_{ff} per unit volume radiated as bremsstrahlung

$$P_{ff} = D (k\,T\,\chi_H/h^2)^{1/2}\, n_e \sum_{(Z)} n_i\, Z^2 \cong 1.5 \cdot 10^{-32} (k\,T)^{1/2}\, n_e \sum_{(Z)} n_i\, Z^2 \qquad (\mathrm{W/cm^3}), \quad (24)$$

where $k\,T$ is in eV.

[23] J. L. Bobin, F. Delobeau, G. de Giovanni, C. Fauquignon, and F. Floux, Nucl. Fusion **9**, 115 [1969].

[24] G. Elwert, Z. Naturforsch. **9 a**, 637 [1954].

b) Recombination Radiation

Besides the change of energy within the continuum a free electron has the possibility to go into a bound state. The radiation emitted during this process is called recombination radiation. The intensities of bremsstrahlung and recombination radiation are of the same order of magnitude if the thermal energy of the free electrons is less than the binding energy. The power $P_{fb}(\nu)$ emitted as recombination radiation per unit volume and unit frequency interval is given in Ref. [24]

$$P_{fb}(\nu) = D\, n_e (\chi_H/k\,T)^{3/2} \sum_{(Z)} n_i(Z)\ f_1 \sum_{(\bar{n})} (\chi_{\bar{n}}/\chi_H)^2\ (\zeta_{\bar{n}}/\bar{n})\ \exp\{(\chi_{\bar{n}} - h\,\nu)/k\,T\}\,, \qquad h\,\nu > \chi_{\bar{n}}. \tag{25}$$

D is the constant given in Eq. (23), $n_i(Z)$ is the density of ions with the charge Z and $\chi_{\bar{n}}$ is the ionization potential of an ion with the charge $(Z-1)$ for electrons in the $\bar{n}$-th shell. $\bar{n}$ is the main quantum number and $\zeta_{\bar{n}}$ the number of unoccupied sites in the $\bar{n}$-th shell. f_1 represents the gaunt correction for free-bound transitions which will be neglected in our estimate.

In the frequency range $h\,\nu > \chi_{\bar{n}}$ recombination radiation and bremsstrahlung show the same spectral behaviour. At $h\,\nu = \chi_{\bar{n}}$ the recombination radiation spectrum shows a discontinuity, since this case corresponds to the capture of a resting free electron by the ion. The total power emitted per unit volume is obtained by integrating Eq. (25) for $h\,\nu > \chi_{\bar{n}}$

$$P_{fb} = D\, n_e (\chi_H^3/h^2\,k\,T)^{1/2} \sum_{(Z)} n_i(Z) \sum_{(\bar{n})} (\chi_{\bar{n}}/\chi_H)^2\ (\zeta_{\bar{n}}/\bar{n}) = 2\cdot 10^{-31} (k\,T)^{-1/2}\, n_e \sum_{(Z)} n_i(Z) \sum_{(\bar{n})} (\chi_{\bar{n}}/\chi_H)^2\ (\zeta_{\bar{n}}/\bar{n}) \quad (\mathrm{W/cm^3}) \tag{26}$$

where $k\,T$ is in eV. From Eq. (26) we see that the power of the recombination radiation may exceed the bremsstrahlung at low temperatures and high ionization potentials.

c) Line Radiation

If there are ions with one or more bound electrons present in the plasma, we also have to consider the excitation of these electrons and the corresponding line radiation. The power P_l emitted as line radiation per unit volume is given in Ref. [25]

$$P_l = 4\sqrt{\pi}\,\alpha\, a_0^2\, c\, \chi_H (\chi_H/k\,T)^{1/2}\, n_e \sum_{(Z)} n_i(Z)\ \exp\{-E(Z)/k\,T\}$$
$$= 3.5\cdot 10^{-25} (k\,T)^{-1/2}\, n_e \sum_{(Z)} n_i(Z)\ \exp\{-E(Z)/k\,T\} \quad (\mathrm{W/cm^3}) \tag{27}$$

omitting any reabsorption (optically thin plasmas). The summation has to be made over all ions with one or more bound electrons. $E(Z)$ is the corresponding excitation energy. Values for $E(Z)$ are given in Ref. [26].

d) Radiation Powers in a Laser Produced Plasma

Before we estimate the radiation losses in a laser produced plasma we have to evaluate the ion densities $n_i(Z)$ for different ionization stages. To do this we use the corona model by ELWERT [27] where we assume an equilibrium between the ionization and recombination processes. In Table 1 the relative population of the different ionized states at three temperatures are compiled for hydrogen, lithium and carbon plasmas. At 100 eV the calculated values for a carbon plasma are in good agreement with the experimentally determine populations in a laser produced carbon plasma [2].

$k\,T$ (eV)	H	Li^{2+}	Li^{3+}	C^{4+}	C^{5+}	C^{6+}
100	1	$2\cdot 10^{-4}$	1	0.36	0.56	0.08
200	1	$5\cdot 10^{-5}$	1	10^{-4}	0.03	0.97
300	1	$2\cdot 10^{-5}$	1	10^{-5}	0.01	0.99

Table 1. Relative population of the different ionization stages for H, Li, and C calculated from the corona model.

	$k\,T$ (eV)	$P_{ff}\cdot V\,(W)$	$P_{fb}\cdot V\,(W)$	$P_l\cdot V\,(W)$	$P\cdot V\,(W)$	$P_L\,(W)$
LiD	100	$3.8\cdot 10^4$	$8.0\cdot 10^4$	—	$1.2\cdot 10^5$	$5.0\cdot 10^7$
	200	$5.4\cdot 10^4$	$5.7\cdot 10^4$	—	$1.1\cdot 10^5$	$2.0\cdot 10^8$
	300	$6.5\cdot 10^4$	$4.5\cdot 10^4$	—	$1.1\cdot 10^5$	$5.0\cdot 10^8$
C	100	$2.0\cdot 10^4$	$5.0\cdot 10^4$	$5.0\cdot 10^6$	$5.0\cdot 10^6$	$2.0\cdot 10^7$
	200	$1.5\cdot 10^5$	$6.0\cdot 10^5$	10^6	$1.7\cdot 10^6$	10^8
	300	$1.9\cdot 10^5$	$5.0\cdot 10^5$	$3.5\cdot 10^5$	$8.5\cdot 10^5$	$2.5\cdot 10^8$

Table 2. Radiated power from a laser produced plasma. ff: bremsstrahlung, fb: recombination radiation, l: line radiation, and L: laser. $P\cdot V$ is the total radiated power. Plasma volume $V = 10^{-7}$ cm³.

[25] H. R. GRIEM, Plasma Spectroscopy, McGraw-Hill Co., New York 1964.

[26] LANDOLT-BÖRNSTEIN, Atom- und Molekularphysik, 1. Teil, Atome und Ionen, Springer-Verlag, Berlin 1950.

In Table 2 values for the power emitted by the three different radiation mechanisms in a laser produced plasma are given for three different temperatures. We assumed a radiating volume V of 10^{-7} cm^3 which is a representative value for the volume of the heating zone. The electron densities were taken from the expression given in Eq. (8). $P\,V$ is the total power of radiation emitted by the plasma. For comparison the laser power P_L which is necessary to heat the plasma up to the indicated temperature is given.

We see from this comparison that in general radiation losses may be neglected. Only at low electron temperatures and with targets with elements of high Z losses due to the line radiation are considerable.

[27] G. Elwert, Z. Naturforsch. **7 a**, 432 [1952].

Temperature and Expansion Energy of Laser Produced Plasmas

II. Experiments

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Experiments on laser produced plasmas with light intensities between $2\cdot10^{11}$ and $5\cdot10^{12}$ W/cm^2 are reported. Measurements of the electron temperature and the expansion energy were performed. In the case of LiD targets the highest temperature observed was 200 eV. Using carbon targets a maximum temperature of 330 eV was measured. The corresponding expansion energies reach values as high as 13 keV. The experimental results are in good agreement with a stationary, hydrodynamic theory. We conclude from our data that in our LiD and C plasmas ions and electrons have the same temperature.

The production of plasmas by irradiating plane, solid targets with intense light pulses has been investigated experimentally by a number of authors (l. c. [1–7]). The main interest of these studies is to determine plasma parameters such as electron density and temperature, ion expansion energy, and total number of particles produced during the interaction. We wish to report results obtained by irradiating plane, solid targets of lithium deuteride [8] and carbon with a powerful giant pulse ruby laser. The experimentally determined values for electron temperature and ion expansion energy will be compared with the results derived from the theory presented in a previous paper [9] (later referred to as A). In Section I we shall summarize the most important results derived in A. Our experimental set up and the results are presented in Sections II and III respectively. In the Appendix, a detailed discussion is presented on the X-ray method which was used for determining the electron temperature.

I. Theoretical Remarks

In A) we developed a hydrodynamic model describing the plasma production as a steady state process. Taking into account the finite dimension of the focal spot we were able to divide the plasma in three regions: a) the undisturbed target, b) a zone of one-dimensional plasma flow extending from the target surface out to a distance R (R ist the focal spot radius), and c) a zone of three-dimensional plasma flow (see Fig. 1 in A). It was shown that during the simultaneous heating and expansion of the plasma the flow velocity u reaches the local velocity of sound at the point $x=R$. Considering an energy balance between the incoming light flux and the thermal- and kinetic energy transported by the plasma flow, we were able to determine the highest plasma temperature $k\,T_1$, the total number of charges N produced during the interaction, and ion expansion energy E as a function of the incident light

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[9] H. Puell, Z. Naturforsch. **25 a**, 1807 [1970]; preceding publication.